$$\therefore\ (a^2+x^2)\frac{dy}{dx}=C;\ dy=\frac{Cdx}{a^2+x^2};\ y=\frac{C}{a}\tan^{-1}\frac{x}{a}+C'.$$

Also solved by S. A. Corey, M. E. Graber, G. W. Greenwood, J. O. Mahoney, E. L. Rich, J. Scheffer, J. E. Sanders, and G. B. M. Zerr.

GEOMETRY.

268. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Find, without the aid of trigonometry, the side of an inscribed regular polygon of $2n$ sides, if the side of an inscribed regular polygon of n sides is 16 feet. [Wentworth's *Plane Geometry*, Revised Edition, problem 512, page 244.]

Solution by M. R. BECK, Cleveland, Ohio.

Let AB be the chord=16. From the center O draw the radius $r=OC$, perpendicular to AB at D. Draw the chord BC=the side of an inscribed regular polygon of $2n$ sides.

In $\triangle OBD$, $r^2=(r-DC)^2+64$, and therefore $DC=r-\sqrt{(r^2-64)}$. Also in $\triangle CDB$, $BC^2=DC^2+64$. Substituting, $BC=\sqrt{\{2r[r-\sqrt{(r^2-64)}]\}}$.

Also solved by P. S. Berg.

269. Proposed by J. SCHEFFER, A. M.

Find the area of a segment, if the chord of the segment is 10 feet, and the radius of the circle is 16 feet.

Solution by P. S. BERG, Larimore, N. D.

Since $16-\sqrt{(16^2-5^2)}=.8014$, height of segment, then

$$\frac{(.8014)^3}{2\times10}+\tfrac{2}{3}\times.8014\times10=5.36 \text{ square feet, area of segment.}$$

Also solved by G. W. Greenwood, M. E. Graber, A. H. Holmes, D. B. Northrup, J. Edward Sanders, and G. B. M. Zerr.

270. Proposed by F. R. HONEY, Ph. B., Hartford, Conn.

What portion of the heavens is always invisible to an observer whose latitude is given?

Solution by J. SCHEFFER, A. M., Hagerstown, Md.

That portion of the heavens which is never visible to an observer in a certain latitude is that cut off by the circle of perpetual occultation. Therefore, if ϕ represents the latitude of the observer, the ratio of the ever invisible portion of the celestial vault to the whole vault is manifestly $\sin^2\frac{1}{2}\phi$.

Also solved by A. H. Holmes, and the Proposer.

271. Proposed by W. J. GREENSTREET, M. A., Stroud, England.

Two equal concentric ellipses have their axes at an angle θ. Find the area of the quadrilateral circumscribing both, in terms of θ and the semi-axes.

Solution by G. W. GREENWOOD, M. A., McKendree College. Lebanon, Ill.

Denote the equation of one ellipse by $x^2/a^2+y^2/b^2=1$. The quadrilateral circumscribing both is evidently a rectangle whose sides are respectively parallel to the common chords. Their equations are, therefore,

$$x\cos\tfrac{1}{2}\theta+y\sin\tfrac{1}{2}\theta=P_1\,; \quad -x\sin\tfrac{1}{2}\theta+y\cos\tfrac{1}{2}\theta=P_2\,; \quad \text{where} \quad P_2^2=a^2\cos^2\tfrac{1}{2}\theta+b^2\sin^2\tfrac{1}{2}\theta\,;$$
$$P_2^2=a^2\sin^2\tfrac{1}{2}\theta+b^2\cos^2\tfrac{1}{2}\theta.$$

The required area$=4P_1P_2=4\sqrt{[(a^2-b^2)\sin^2\tfrac{1}{2}\theta\cos^2\tfrac{1}{2}\theta+a^2b^2]}$.

Also solved by A. H. Holmes, A. S. Hawkesworth, and G. B. M. Zerr.

272. Proposed by R. D. CARMICHAEL, Hartselle, Ala.

A point A revolves with uniform speed in a circle. A point B revolves around A, at a uniform distance from it, with the same angular velocity, but in the opposite direction. Determine the locus of B.

Solution by G. W. GREENWOOD, M. A., McKendree College, Lebanon, Ill.

Denote the center of the fixed circle by O, and take Ox through A and B at a time when these points are collinear, A lying between O and B. Let A move counter clockwise. Then $x=(a+b)\cos\theta$, $y=(a-b)\sin\theta$.

$\therefore \dfrac{x^2}{(a+b)^2}+\dfrac{y^2}{(a-b)^2}=1$, where a, b are the radii of the fixed and moving circles, respectively.

Also solved by A. H. Holmes, A. S. Hawkesworth, D. B. Northrup, G. B. M. Zerr, and Proposer.

GROUP THEORY.

11. Proposed by DR. SAUL EPSTEEN, University of Colorado, Boulder.

Find the six-parameter continuous group which leaves invariant the surface of second order $x_1x_2-x_3x_4=0$.

Solution by the PROPOSER.

To every transformation of this group defined as below, correspond four different sets of values of the parameters a, b, ... In particular to the identical transformation corresponds $a=d=\pm1$, $\alpha=\delta=\pm1$, $b=c=\beta=\gamma=0$.

The group is

$$x'_1=\alpha(ax_1+bx_4)+\beta(ax_3+bx_2),$$
$$x'_2=\gamma(cx_1+dx_4)+\delta(cx_3+dx_2),$$
$$x'_3=\gamma(ax_1+bx_4)+\delta(ax_3+bx_2),$$
$$x'_4=\alpha(cx_1+dx_4)+\beta(cx_3+dx_2).$$